A CAD-Oriented Method for Noise Figure Computation of Two-Ports with Any Internal Topology

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Abstract —**A method for computer-aided noise analysis and an algorithm for the noise parameter computation of two-ports with any internal topology are discussed. The approach is applicable to circuits which are composed of any number of passive linear multiports and active linear two-port devices. The noise analysis is based on the scattering matrix description for circuit elements and wave representation for noise.**

I. Introduction

THE THEORY of noisy, linear cascaded two-ports is well established and has provided the basis for optimizing the performance of low-noise amplifiers. This theory has been based on the classical work by Rothe and Dahlke [1], who use an impedance or admittance representation of noise parameters. The representation of the noise of microwave networks in terms of power waves is also used very effectively [2]–[4]. Transformation formulas for the four noise parameters given in [5] are useful for noise analysis of circuits that may be represented by series, parallel, and cascaded connections of two-ports. The method for noise analysis proposed in [6] has the same restrictions. The two-port which is to be analyzed is viewed as an interconnection of two-ports only.

Expressions are also known for the noise figure of cascaded structures with series or parallel feedback [7], [8] and for distributed amplifiers [9], [10]. These relations are only valid for preselected topologies. They cannot be implemented into general-purpose computer programs for noise analysis of circuits with any topology.

It is the purpose of this paper to present a computer-aided noise analysis method for linear two-ports with absolutely general internal topology.

Instead of an admittance matrix and a current description for circuits and noise which are used in the method described in [11] and [12], a scattering matrix and a wave representation are presented. At microwave frequencies, a treatment of noise in terms of waves is more attractive. The ingoing and outgoing noise waves A_i and B_i are given by

$$A_i = \frac{V_i + Z_i I_i}{2\sqrt{\mathrm{Re}(Z_i)}} \qquad B_i = \frac{V_i - Z_i^* I_i}{2\sqrt{\mathrm{Re}(Z_i)}} \tag{1}$$

where V_i and I_i are the noise voltage and the noise current flowing into the ith port of a circuit and Z_i is the reference impedance of the port.

Equation (1) is identical to the standard definition of power waves for sinusoidal signals [4].

In the noise analysis of circuits containing multiports, one noise wave source is placed at each circuit port. These noise wave sources represent noise generated in each element (multiport) of the circuit. The method makes it possible to effectively compute the noise figure and four noise parameters of the overall circuit. The information about minimum noise figure and noise-matching conditions is an important advantage of the approach. Thanks to the connection scattering matrix formalism used for a circuit description, the noise analysis method presented is more compatible with CAD software [12]–[16] than the method described in [17].

II. Method of Analysis

In the analysis, it is assumed that each linear noisy network may be represented as the interconnection of lossy passive multiports which introduce only thermal noise and noisy active two-ports. Each linear element in the circuit may be represented by its noiseless equivalent having the same scattering matrix S as the original network. As is shown in Fig. 1, noise generated in an element is

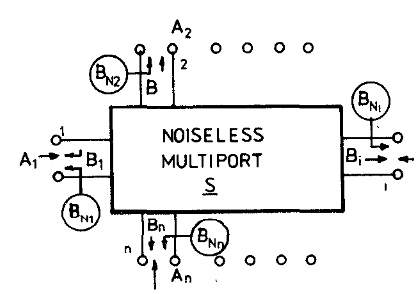

Fig. 1. Wave representation of noise in a multiport.

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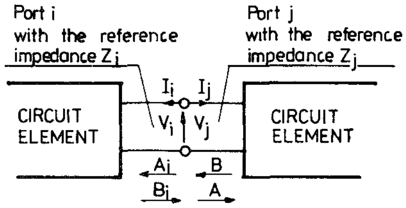

Fig. 2. Constraints imposed by connections between adjacent ports.

represented by mutually correlated noise wave sources, one source at each port.

In matrix notation, a set of linear equations which relate complex amplitudes of noise waves at ports of a circuit element has the from [18]

$$\boldsymbol{B}^{(k)} = \boldsymbol{S}^{(k)}\boldsymbol{A}^{(k)} + \boldsymbol{B}_N^{(k)} \tag{2}$$

where $\boldsymbol{S}^{(k)}$ is the scattering matrix of the kth element, $\boldsymbol{A}^{(k)}$ and $\boldsymbol{B}^{(k)}$ are vectors of incident and reflected noise waves at its ports, and $\boldsymbol{B}_N^{(k)}$ is a vector of mutually correlated noise wave sources which represent noise generated in the element. The noise waves from these sources radiate out of the ports and they do not depend on incident noise waves $\boldsymbol{A}^{(k)}$.

Fig. 2 presents a general circuit composed of m elements (multiports) connected together by their ports.

Considering all m elements of the circuit, we have a set of linear equations whose matrix form is

$$\boldsymbol{B} = \boldsymbol{S}\boldsymbol{A} + \boldsymbol{B}_N \tag{3}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}^{(1)} \\ \boldsymbol{A}^{(2)} \\ \vdots \\ \boldsymbol{A}^{(k)} \\ \vdots \\ \boldsymbol{A}^{(m)} \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}^{(1)} \\ \boldsymbol{B}^{(2)} \\ \vdots \\ \boldsymbol{B}^{(k)} \\ \vdots \\ \boldsymbol{B}^{(m)} \end{bmatrix} \quad \boldsymbol{B}_N = \begin{bmatrix} \boldsymbol{B}_N^{(1)} \\ \boldsymbol{B}_N^{(2)} \\ \vdots \\ \boldsymbol{B}_N^{(k)} \\ \vdots \\ \boldsymbol{B}_N^{(m)} \end{bmatrix} \tag{4}$$

and

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}^{(1)} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}^{(2)} & & & \boldsymbol{0} \\ \vdots & & \ddots & & \vdots \\ \boldsymbol{0} & & & \boldsymbol{S}^{(k)} & \boldsymbol{0} \\ \vdots & & & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \cdots & \boldsymbol{S}^{(m)} \end{bmatrix} \tag{5}$$

The connections between the m elements impose constraints on the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ which can be represented as a matrix equation.

$$\boldsymbol{B} = \Gamma \boldsymbol{A} \tag{6}$$

where Γ is the connection matrix. In fact, incident and reflected noise waves at ports i and j connected together must satisfy the following relation (see Fig. 2):

$$\begin{bmatrix} b_i \\ b_j \end{bmatrix} = \frac{1}{Z_i + Z_j} \cdot \begin{bmatrix} Z_j - Z_i^* & 2\sqrt{\mathrm{Re}(Z_i)\mathrm{Re}(Z_j)} \\ 2\sqrt{\mathrm{Re}(Z_i)\mathrm{Re}(Z_j)} & Z_i - Z_j^* \end{bmatrix} \begin{bmatrix} a_i \\ a_j \end{bmatrix} \tag{7}$$

where Z_i and Z_j are the reference impedances of the connected ports. The above relation defines elements of the connection matrix Γ corresponding to a pair of the connected ports.

It is assumed in the analysis that for all pairs of the connected ports

$$Z_i = Z_j^*. \tag{8}$$

This means that all port connections in the analyzed circuit are nonreflecting; that is, $a_i = b_j$ and $a_j = b_i$. In such a case the elements of the connection matrix Γ are all zero except the 1's in the entries corresponding to pairs of adjacent ports.

After elimination of the vector $\boldsymbol{B}$ from (3) and (6) we get

$$\boldsymbol{W}\boldsymbol{A} = \boldsymbol{B}_N \tag{9}$$

where

$$\boldsymbol{W} = \Gamma - \boldsymbol{S} \tag{10}$$

is the connection scattering matrix of the analyzed circuit [13].

Using (9) we are able to get a correlation matrix of the incident noise waves in all circuit ports. Given that

$$\boldsymbol{A} = \boldsymbol{W}^{-1}\boldsymbol{B}_N \tag{11}$$

then

$$\overline{\boldsymbol{A}\boldsymbol{A}^\dagger} = \boldsymbol{W}^{-1}\overline{\boldsymbol{B}_N\boldsymbol{B}_N^\dagger}(\boldsymbol{W}^{-1})^\dagger = \boldsymbol{W}^{-1}\boldsymbol{C}(\boldsymbol{W}^{-1})^\dagger \tag{12}$$

where the bars indicate the statistical averages and the daggers the complex conjugate transpose of the vectors and matrices. In (12)

$$\boldsymbol{C} = \overline{\boldsymbol{B}_N\boldsymbol{B}_N^\dagger} \tag{13}$$

is the correlation matrix of the noise wave sources representing noise generated in all circuit elements.

Because the noise wave sources $\boldsymbol{B}_N^{(i)}$ of an (ith) element are uncorrelated with those of any other circuit element, the correlation matrix $\boldsymbol{C}$ is a block diagonal matrix of the form

$$\boldsymbol{C} = \overline{\boldsymbol{B}_N\boldsymbol{B}_N^\dagger} = \begin{bmatrix} \boldsymbol{C}^{(1)} & \boldsymbol{0} & \cdot & \cdots & \cdot & \cdots & \cdot & \boldsymbol{0} \\ 0 & \boldsymbol{C}^{(2)} & \cdot & & & & & \vdots \\ \vdots & & & \ddots & & & & \vdots \\ & & & & \boldsymbol{C}^{(k)} & & & \\ \vdots & & & & & \ddots & & \boldsymbol{0} \\ \boldsymbol{0} & \cdot & \cdot & \cdots & \cdot & \cdots & \boldsymbol{0} & \boldsymbol{C}^{(m)} \end{bmatrix} \tag{14}$$

in which $\boldsymbol{C}^{(1)}, \boldsymbol{C}^{(2)}, \cdots, \boldsymbol{C}^{(m)}$ are correlation matrices of the noise wave sources of individual circuit elements.

A. Active Two-Ports

Noise generated in an active two-port is represented by the complex waves B_{N1} and B_{N2} radiating from both ports. Their 2×2 correlation matrix

$$C=\overline{B_N B_N^\dagger}=\begin{bmatrix}\overline{B_{N1}B_{N1}^*} & \overline{B_{N1}B_{N2}^*}\\ \overline{B_{N2}B_{N1}^*} & \overline{B_{N2}B_{N2}^*}\end{bmatrix} \tag{15}$$

may be calculated from the following relation:

$$C=\frac{kT\Delta f}{1-|\Gamma_o|^2}\begin{bmatrix}\left[F_{em}+(N-F_{em})|\Gamma_o|^2\right]|S_{11}|^2+2N\operatorname{Re}(\Gamma_o S_{11})+N-F_{em}(1-|\Gamma_o|) & \left[F_{em}+(N-F_{em})|\Gamma_0|^2\right]S_{11}S_{21}^*+N\Gamma_o^*S_{21}^*\\ \left[F_{em}+(N-F_{em})|\Gamma_o|^2\right]S_{11}^*S_{21}+N\Gamma_o S_{21} & \left[F_{em}+(N-F_{em})|\Gamma_o|^2\right]|S_{21}|^2\end{bmatrix} \tag{16}$$

where

$$N=4R_N\cdot G_o \tag{17}$$

and

F_{em} = minimum excess noise figure,
Γ_0 = $\operatorname{Re}\Gamma_0+j\operatorname{Im}\Gamma_o$, optimum reflection coefficient of the signal source,
R_N = noise resistance,
G_o = real part of the optimum source admittance,
S_{ij} = $(i,j=1,2)$ scattering parameters of a two-port.

F_{em}, $\Gamma_0=\operatorname{Re}\Gamma_0+j\operatorname{Im}\Gamma_0$, and N are a set of noise parameters which must be obtained through measurements [19]–[21]. The correlation matrix C may also be expressed by any other set of active two-port noise parameters, for example T_m, R_o, X_o, R_N or T_M, G_o, B_o, G_N [6], [22].

B. Passive Multiports

Lossy passive multiports generate only thermal noise. It is represented again by the complex waves B_{N1}, $B_{N2},\cdots,B_{Nn}$, where n is the number of ports in a multiport. The $n\times n$ correlation matrix C of these noise sources is given by [23]

$$\begin{aligned}C&=\left[\overline{B_{Ni}B_{Nj}^\dagger}\right],\qquad i,j=1,2,\cdots,n\\ &=kT\Delta f(I-SS^\dagger)\end{aligned} \tag{18}$$

where
k Boltzmann's constant,
T physical temperature of the multiport,
I identity matrix,
S scattering matrix of the multiport.

The quantity $(I-SS^\dagger)$ is called the noise distribution matrix because it describes how the thermal noise power generated in the multiport is distributed over its ports.

It should be mentioned here that, in general, passive elements in a microwave circuit can have different physical temperatures.

III. An Algorithm for the Noise Figure Computation of a General Multiport Circuit

To compute the noise figure, we will assume that the output port load impedance is noise free. Under such a condition, the noise figure of the circuit at frequency f is given by [24]

$$F=1+\frac{P_{N\,\mathrm{int}}}{P_{NS}} \tag{19}$$

where

$P_{N\,\mathrm{int}}$ (available) active noise power at the output port of the circuit arising from the noise sources acting within the circuit,
P_{NS} (available) active noise power at the output port of the circuit arising from the equivalent thermal noise source of the input port termination.

If r is the number of the load impedance port of the analyzed circuit, then the active noise power at the output port is

$$P_N=\left(\overline{AA^\dagger}\right)_{rr}\left(1-|S_{rr}|^2\right) \tag{20}$$

where $(\overline{AA^\dagger})_{rr}$ is the rth diagonal element of the correlation matrix $\overline{AA^\dagger}$,

$$S_{rr}=\frac{Z_L-Z_r^*}{Z_L+Z_r} \tag{21}$$

is the reflection coefficient of the output port load, Z_L is the load impedance, and Z_r is the reference impedance of the load impedance port. Noise powers $P_{N\,\mathrm{int}}$ and P_{NS} can therefore be computed using the simple matrix multiplications described by (12).

The following steps carried out by the numerical algorithm are as follows:

a) The connection scattering matrix W of the analyzed circuit it built and computed.
b) The noise correlation matrices of the individual passive and active circuit elements are found. The noise correlation matrix C defined by (13) is built and computed. Two diagonal elements of C relative to ports belonging to the signal source impedance and to the load impedance are set to zero.

$$C=\begin{array}{c}\\ \\ p\\ \\ r\\ \\ \end{array}\!\!\begin{array}{c}\qquad\qquad\qquad\quad p\qquad\qquad\quad r\\ \begin{bmatrix}C^{(1)} & & \vdots & & \vdots & \\ & C^{(2)} & \vdots & & \vdots & \\ \cdots & \cdots & 0 & & \vdots & \\ & & & C^{(k)} & \vdots & \\ \cdots & \cdots & \cdots & \cdots & 0 & \\ & & & & & C^{(m)}\end{bmatrix}\end{array}$$

p: signal generator port number
r: load impedance port number.

Both zeros represent no noise power generated by signal source and load impedances.

c) Relative to the load impedance port diagonal element of the matrix $\overline{AA^\dagger}$ is computed using (12). A value of this element multiplied by $(1-|S_{rr}|^2)$ equals $P_{N\,\mathrm{int}}$.

d) The noise correlation matrix C defined by (14) is built and computed once again, this time for the case where noise in the circuit originates from the equivalent thermal noise source of the input port termination only. It means that all elements of the matrix C must be equal to zero, except an element corresponding to the input port termination. According to (18) this element is

$$C_{pp} = kT\left(1-|S_{pp}|^2\right) \tag{22}$$

where

$$S_{pp} = \frac{Z_s - Z_p^*}{Z_s + Z_p} \tag{23}$$

is the reflection coefficient of the input port termination (signal source port). Z_s is the impedance of the input port termination (signal source impedance), and Z_p is the reference impedance of the signal source port:

$$\begin{array}{cc} & p \\ C = p & \begin{bmatrix} \mathbf{0} & & \vdots & & & \\ & \mathbf{0} & \vdots & & & \\ \cdots & \cdots & C_{pp} & & & \\ & & & \mathbf{0} & & \\ & & & & \mathbf{0} & \\ & & & & & \mathbf{0} \end{bmatrix} \end{array}$$

p: signal source port number.

e) Relative to the load impedance port diagonal element of the matrix $\overline{AA^\dagger}$ is computed using (12). This time the value of this element multiplied by $(1-|S_{rr}|^2)$ equals P_{NS}.

f) The noise figure F is found from (19) using results obtained in steps c and e.

Note that the matrix W in (12) is the connection scattering matrix of the analyzed circuit, so that a conventional circuit analysis may be carried out together with the noise figure calculation. This allows a simultaneous optimization of an absolutely general amplifier topology with respect to both noise figure and any of the conventional circuit functions. The inverse matrices W^{-1} and $(W^{-1})^\dagger$ of the connection scattering matrix W can be computed very effectively using sparse matrix technique [14], [15].

IV. An Algorithm for Computation of the Four Noise Parameters of a General Multiport Circuit

To compute the four noise parameters related to the input and output ports of a general multiport circuit, we assume that the reflection coefficient S_{pp} of the input port termination (equation (23)) and the reflection coefficient S_{rr} of the output port load (equation (21)) are both zero. We assume also that both these terminations are noise free. Under such assumptions the noise waves at the input and output ports can be computed using (12).

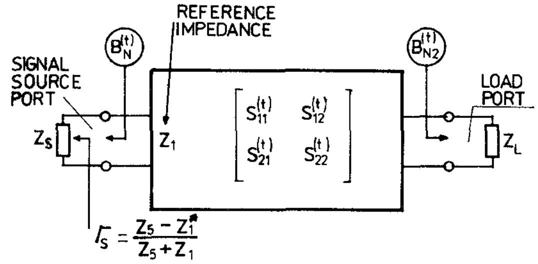

Fig. 3. Outgoing noise waves $B_{N1}^{(t)}$ and $B_{N2}^{(t)}$ at the input and output ports of the overall circuit.

The actual steps carried out by the numerical algorithm are listed below:

a) The connection scattering matrix W of the analyzed circuit is built and computed. Diagonal elements $W_{pp} = -S_{pp}$ and $W_{rr} = -S_{rr}$ of W must be set to zero (p is the signal source port number, r the load impedance port number).

b) The noise correlation matrices of the individual passive and active circuit elements are found. The noise correlation matrix C defined by (13) is built and computed. Two diagonal elements of the matrix C relative to ports belonging to the signal source impedance and to the load impedance are set to zero.

c) The three elements $(\overline{AA^\dagger})_{pp}$, $(\overline{AA^\dagger})_{rr}$, and $(\overline{AA^\dagger})_{pr}$ (or $(\overline{AA^\dagger})_{rp}$) of the matrix $\overline{AA^\dagger}$ are computed using (12).

The correlation matrix of the outgoing noise waves $B_{N1}^{(t)}$ and $B_{N2}^{(t)}$ at the input and output ports of the overall network is given by (see Fig. 3)

$$C^{(t)} = \begin{bmatrix} C_{11}^{(t)} & C_{12}^{(t)} \\ C_{12}^{(t)} & C_{22}^{(t)} \end{bmatrix} = \begin{bmatrix} \overline{B_{N1}^{(t)} \cdot B_{N1}^{(t)*}} & \overline{B_{N1}^{(t)} \cdot B_{N2}^{(t)*}} \\ \overline{B_{N2}^{(t)} \cdot B_{N1}^{(t)*}} & \overline{B_{N2}^{(t)} \cdot B_{N2}^{(t)*}} \end{bmatrix} = \begin{bmatrix} \overline{(AA^\dagger)}_{pp} & \overline{(AA^\dagger)}_{pr} \\ \overline{(AA^\dagger)}^*_{pr} & \overline{(AA^\dagger)}_{rr} \end{bmatrix}. \tag{24}$$

Once the elements of the noise correlation matrix $C^{(t)}$ are known, the noise figure F can be determined. The noise figure is given by

$$F = 1 + \frac{C_{11}^{(t)} \left| \dfrac{\Gamma_s S_{21}^{(t)}}{1 - S_{11}^{(t)}\Gamma_2} \right|^2 + C_{22}^{(t)} + 2\,\mathrm{Re}\left\{ C_{12}^{(t)} \dfrac{S_{21}^{(t)}\Gamma_s}{1 - S_{11}^{(t)}\Gamma_s} \right\}}{kT_0 \Delta f \dfrac{|S_{21}^{(t)}|^2}{|1 - S_{11}^{(t)}\Gamma_s|^2}\left(1 - |\Gamma_s|^2\right)} \tag{25}$$

where $S_{11}^{(t)}$ and $S_{21}^{(t)}$ are the scattering parameters of the overall network, and

$$\Gamma_s = \frac{Z_s - Z_1^*}{Z_s + Z_1} \tag{26}$$

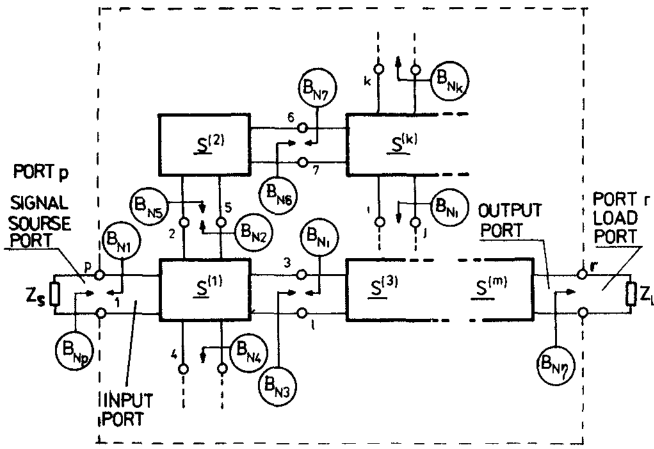

Fig. 4. Equivalent circuit of a multiport network with noiseless elements and noise wave sources at each port.

is the reflection coefficient of the signal source impedance with respect to the input port reference impedance.

Using the $C^{(t)}$ matrix elements, any set of noise parameters of the overall network may be calculated, for example F_{em}, Γ_o, and N:

$$F_{em}=\frac{1}{2}\left[C_{22}^{(t)}\frac{1-|S_{11}^{(t)}|^2}{|S_{21}^{(t)}|^2}-C_{11}^{(t)}+2\,\mathrm{Re}\left\{C_{12}^{(t)}\frac{S_{11}^{(t)*}}{S_{21}^{(t)*}}\right\}+N\right] \tag{27}$$

$$\Gamma_o=\frac{2\left(C_{12}^{(t)*}\dfrac{1}{S_{21}^{(t)}}-C_{22}^{(t)}\dfrac{S_{11}^{(t)*}}{|S_{21}^{(t)}|^2}\right)}{C_{22}^{(t)}\dfrac{1+|S_{11}^{(t)}|^2}{|S_{21}^{(t)}|^2}+C_{11}^{(t)}-2\,\mathrm{Re}\left\{C_{12}^{(t)}\dfrac{S_{11}^{(t)*}}{S_{21}^{(t)*}}\right\}+N} \tag{28}$$

$$N=\left\{\left[C_{22}^{(t)}\frac{1+|S_{11}^{(t)}|^2}{|S_{21}^{(t)}|^2}+C_{11}^{(t)}-2\,\mathrm{Re}\left\{C_{12}^{(t)}\frac{S_{11}^{(t)*}}{S_{21}^{(t)*}}\right\}\right]^2 -4\left|C_{12}^{(t)*}\frac{1}{S_{21}^{(t)}}-C_{22}^{(t)}\frac{S_{11}^{(t)*}}{|S_{21}^{(t)}|^2}\right|^2\right\}^{1/2}. \tag{29}$$

This time the noise figure of the overall circuit is given by

$$F=1+F_{em}+N\frac{|\Gamma_s-\Gamma_o|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_o|^2)}. \tag{30}$$

The overall scattering parameters $S_{11}^{(t)}$ and $S_{21}^{(t)}$ used in (25) and (27)–(29) can be computed from the equation

$$\boldsymbol{W}\boldsymbol{a}=\boldsymbol{b}_s \tag{31}$$

where $\boldsymbol{W}$ is the connection scattering matrix given by (10), $\boldsymbol{a}$ is a vector of incident power waves, and $\boldsymbol{b}_s$ is a vector of the impressed power waves of the independent sinusoidal signal sources [13]–[16].

Connecting the input port of an analyzed multiport circuit (see Fig. 4) to a matched signal source ($S_{pp}=0$) with the impressed wave $b_{sp}=1$ and the output port to a matched load ($S_{rr}=0$), we get a case in which the overall scattering parameters $S_{11}^{(t)}$ and $S_{21}^{(t)}$ coincide with the waves a_r and a_p:

$$S_{11}^{(t)}=a_r\big|_{a_1=1} \qquad S_{21}^{(t)}=a_t\big|_{a_1=1}.$$

The condition $a_1=1$ is imposed by the signal source connected to the input port.

V. Conclusion

The noise analysis concept presented in this paper is applicable to multiport circuits with any topology. Therefore it is applicable to most networks occurring in microwave practice. The set of noise parameters which can be calculated by the method includes the noise figure, the correlation matrix of the outgoing noise waves at the input and output ports, or the minimum noise figure, the optimum signal source reflection coefficient, and the parameter N.

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